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Sidelights on the Cardan-Tartaglia Controversy

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Humanism and History of Mathematics

Edited by
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Sidelights on the Cardan-Tartaglia Controversy

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I.

There is quite a difference in the frame of mind which comes with the answer to a problem only vaguely defined and lying in an uncharted field, like the invention of the differential calculus, or with a discovery that comes undivined like a flash of lightning from some human mind, like the invention of logarithms,—and the reaction that greets the answer to a problem posed to the world for centuries when that answer arrives, two thousand years in the coming.

The solution of the cubic had presented itself to the human mind as an intellectual problem already in the fifth century B. C.; it became a scientific need in Archimedes' calculation on floating bodies in the third century B. C.; it confronted the Arab astronomers in the Middle Ages. And now it was solved! The first of "the three unsolved problems of antiquity" to be solved.

It produced a great impression. How great, one can gauge from the fact that all respectable texts on algebra for the next 200 years gave long chapters and discussions to the cubic equation. The influence of the discovery must be gauged not only by its mathematical fruitfulness, which after all did not prove to be so very great, but by the stimulus it gave to study, the courage it gave the human mind to soar into the unknown and "make the impossible possible."

The main events leading up to the discovery of a general solution of the cubic equation and the ensuing controversy are given in the various histories of mathematics. But there are illuminating sidelights in this unique controversy,—documentary, anecdotal, biographical,—which do not lend themselves to recording in a well-balanced

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history of mathematics but which are of absorbing interest to the members of the guild of mathematicians. There are the many source materials, for one thing; from some of these we shall quote extracts. There is the language and symbolism, or lack of it, of the algebra prior to Vieta, Stevin, and Descartes. And then there is the exposition of the status of algebraic theory before the monumental works of Cardan and Tartaglia.

The 16th century custom of scientific "duels" and public disputations were a joint inheritance from the philosophical disputations of the Schoolmen and the tournaments of the knights. A chief canon of combat was that no one should propose a question or problem that he himself could not solve. The outward forms were modeled somewhat after the contests of arms,—challenge, response, witnesses, judges, keeper of the stakes, etc.

Public challenges were given, not only for acquiring glory and prestige, but also for making a living. The vanquished, honor lost, had no more pupils; while the victor, heralded and fêted, would be called to various cities to teach and lecture. Consequently, many inventors guarded their secrets. There must have been many discoveries lost to the world due to this custom. Tartaglia himself died while still writing on his algebra and before reaching his contemplated climax on his solution of the cubic; and except for the premature publication of it by Cardan and Tartaglia's accusation in the *Quesiti* his solution might have died with him.

II.

The *Dramatis Personae* of the celebrated controversy were five: Zuanne de Tonini da Coi, Antonio Maria Fior, Girolamo Cardano, Nicolo Tartaglia, and Ludovico Ferrari. The time: 1530 to 1548. Place: Pavia, Padua, Bologna, Milano, Brescia, Venice, the centers of art and learning in Renaissance Italy.

The first two were minor characters and little is known about them outside their connection with this controversy; they were messengers, links, as it were, to bring about action between the other three. *Zuanne da Coi* (sometimes called Giovanni dal Colle) was a teacher in Brescia interested in mathematics from the standpoint of problem solving. *Antonio Maria Fior* (sometimes written Floridus and Del Fiore) flitted about from place to place, causing battle and disturbance; but History will thank him for it. He was an arithmetician, having according to reports no theory knowledge in algebra. He had been a pupil of Scipio Ferro, of whom more later.

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Nicolo Tartaglia was born at Brescia in 1506, died at Venice in 1557. He came from a very poor family, was left fatherless at the age of six, and had only two weeks of formal schooling; but by self-education his powerful mind mastered both the classics and the then known mathematics. He taught mathematics in Verona, Vicenza, Brescia, and, from 1534 or 1535 until his death in 1557, in Venice. His principal mathematical works are: *Nova Scienza* (1557), where he is the first one to discuss the problems of gunnery and fortification mathematically; (2) *Quesiti ed invenzioni diverse* (1546), in nine books, of which the last one deals with algebra; (3) *General Trattato di numeri, e misure* in two volumes (the first published in 1556, the second in 1560) including an arithmetic, a treatise on numbers, and his work on algebra.

Girolamo Cardano (Hieronymus Cardanus or Jerome Cardan) was born at Pavia in 1501, died in Rome in 1576. He received a good university education in Pavia and Padua, having equal zest for medicine and mathematics. Between 1524 and 1550 he taught and practiced medicine, much of the time in Milano; in the same period he studied mathematics assiduously and published many important works. In 1562 he became a university professor at Bologna and in 1570 he moved to Rome to become astrologer to the Pope. He wrote voluminously on many subjects, but in mathematics we mention these: (1) *Practicae Arithmeticae* (1539); (2) *De Regula Aliza* (1540); (3) *Ars Magna* (1545), the first systematic work in algebra up to that time, a text that helped to clarify the principles of algebra and lift the subject out of mere equational problem solving into a theory of equations.

Ludovico Ferrari was born at Bologna in 1522 and died there in 1565. His parents were poor and he came to Cardan's house as an errand boy. He was later allowed to listen to his master's lectures and before long became Cardan's most brilliant pupil. For all his moral lapses and irascible temper, Ferrari was ever loyal to his protector; in fact, looked upon himself as owing his very being to Cardan, designating himself as "suo creato." As far as we know he never published anything independently. What he discovered he let Cardan incorporate into the *Ars Magna*. One of his discoveries was a general solution of the biquadratic equation. For he was able, by using the solution of a cubic already discovered by Tartaglia, to solve the question proposed by Da Coi, namely $x^4+6x^2+36=60x$, succeeding where both Tartaglia and Cardan had failed. Ferrari was only twenty-three years old when the *Ars Magna* was published.

In reading works on algebra from this period the reader must try to divorce from his consciousness many of the ideas and forms he

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has associated with algebra. He will remember that in 1500 there were no imaginary numbers; they did at times make unwelcomed appearances but were not legitimized. Negative numbers did not have operational status and $x^3=px+q$ had a different solution from that of $x^3+px=q$, for instance. The symbols $+$, $-$, $=$, in our sense, did not exist, and our words "plus" and "minus" were not conventionalized. The unknown was variously called *thing, side, cos, res*. Thus Tartaglia's equation ("capitolo") $x^3+3x^2=5$ was "a cube and three censi are equal to five."

Besides Cardan's *Ars Magna* and Tartaglia's *Quesiti* and *General Trattato* we have as source material the six *Cartelli* (letters of challenge) of Ferrari and the six *Risposti* (responses) of Tartaglia. These were sent as printed pamphlets to the mathematicians of Italy.

Being literature of a day it is a wonder that all the twelve bulletins have come down to us. As one might expect, they have their own exciting history. In 1844 Prof. Silvestro Gherhardi owned a volume containing the six Cartelli of Ferrari and the first five Risposti of Tartaglia. In 1848, after a four years' search in all the libraries and old book-stores of the various cities of Italy, Gherhardi finally laid his hands on the missing 6th Risposta in an old book shop in Bologna, and it is the only copy of this Risposta found so far.* Previously all that was known of Tartaglia's 6th letter were citations from Bombelli (1572) and writers living later than Tartaglia by 200 years. In 1858 Gherhardi, meeting with political vicissitudes and exile and in need of money, sold his copy to Libri of London, it being "clipped" on to the other eleven. But firstGherardi was permitted to make an exact copy of the letters "by the hand of Benaducci di Foligno." And that was fortunate; for the copy sent to Libri was lost. So it has been by a slender thread that the last of the twelve letters has reached us. In 1876 the twelve letters were "collected, autographed, and published" by Enrico Giordani in a limited edition of 212 copies under the title *I sei cartelli di matematica disfida di Ludovico Ferrari con sei contro-cartelli in risposta di Nicolo Tartaglia.*†

An additional word concerning Tartaglia's *Quesiti ed invenzioni diverse*. It consists of short, sprightly accounts in dialogue form of problems he discussed with or solved for various people, the first *Quesiti* dated 1521, the last, 1541. Quoting conversations and letters, citing dates, places, and names of interlocutors, many of whom were still living, the book has strong documentary secureness. It is charmingly written, besides. The last of the nine books, comprising 42 *quesiti*, deals with algebra.

**Cartelli* and *Risposta*: Introduction, pp. 9, 12, 15.
†Hereafter referred to as *Cartelli* and *Risposti*.

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III.

The first to give a general solution for a cubic equation was neither Cardan nor Tartaglia. That honor belongs to Scipio del Ferro, professor of mathematics at Bologna.

As late as 1494 Luca Pacioli in his authority-carrying *Summa* had set forth these types of equations as not yet being solved:

$$n = ax + bx^3$$

$$n = ax^2 + bx^3$$

$$n = ax^3 + bx^4.$$

And he intimated that their solutions might not be possible. However, the first of these was solved by Ferro of Bologna.

About all we know of Scipio Ferro is that he was born at Bologna about 1465 and died there in 1526, that he was professor at the University of Bologna from 1496 to 1526, that he had a general solution for $x^3 + px = q$, and that he confided his method to his pupil Antonio Maria Fior. We donot know whether he had derived it himself or found it in an Arab work; whether it was an empirical formula or was the product demonstration. What writings he left must have come into the hands of of his son-in-law Annibale della Nave, who succeeded him in his professorship (1526-1560). But no such writings are extant. Both Cardan and Tartaglia refer to the solution Fior received from Ferro, Tartaglia placing it about 1506, Cardan placing it at about 1514.* It was probably even later.

IV.

Curiously enough, the one who seems to have set the wheels in motion for the final onslaught on the cubic equation was a man of meagre mathematical attainment but of much physical mobility. It was Zuanne de Tonini da Coi. Teaching in Brescia he had, of course, heard of the work of Nicolo of Brescia, now of Verona. In 1530, as one Brixellite to another, with more courage than prudence he proposed to Tartaglia two problems which reduced to solving the equations $x^3 + 3x^2 = 5$ and $x^3 + 6x^2 + 8x = 1000$.

This, the opening chapter in the history of the exciting discovery, is described by Tartaglia in his *Quesiti*,† namely in Quesito XIV. There for the first time we learn that Tartaglia (at this time only 24 years of age) had been dabbling with the cubic.

**Ars Magna*, Nürnberg, 1545, Ch. XI; *Quesito* XXV.
†*Quesiti et inventioni diverse de Nicolo Tartaglia.* At the press of the author, 1554.

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It will give the reader a little of the flavor of the period and give him a peek into one of the interesting books of mathematics to read Tartaglia's own first reference to the attack on the cubic equation.

"QUESITO XIV,

which was propsed to me at Verona by one Maestro Zuanne de Tonini da Coi, who has a school in Brescia, and was brought to me by Messer Antonio da Cellatico in the year 1530.

Maestro Zuanne.—Find a number which multiplied by its root increased by three equals five. Similarly find three numbers such that the second is greater by two than the first and the third is greater by two than the second and where the product of the three is 1000.

N.—M. Zuanne, you have sent me these two questions of yours as something impossible to solve or at least as not being known by me; because, proceeding by algebra, the first leads to the operation on a cube and 3 censi equal to 5, and the second on a cube and 6 censi and 8 cose equal to 1000. [That is, $x^3+3x^2=5$; $x^3+6x^2+8x=1000$]. By F. Luca and others these equations have up to now been considered to be impossible of solution by a general rule. You believe that with such questions you can place yourself above me, making it appear that you are a great mathematician. I have heard that you do this towards all the professors of this science in Brescia, so that they for fear of these your questions do not dare to talk with you; and perhaps they know more about this science than you......................

M. Z.—I understand as much as you have written to me and that you think such cases are impossible,.............................

N.—I do not say such cases are impossible. In fact, for the first case, that of the cube and the censi equal to a number, I am convinced I have found the general rule, but for the present I want to keep it secret for several reasons. For the second, however, that of the cube and censi and cose equal to a number, I confess I have not up till now been able to find a general rule; but with that I do not want to say it is impossible to find one simply because it has not been found up to the present. However, I am willing to wager you 10 ducats against 5 that you are not able to solve with a general rule the two questions that you have proposed to me. And that is something for which you should blush, to propose to others what you yourself do not understand, and to pretend to understand in order to have the reputation of being something great."

That ends the first encounter.

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V.

We now go back a ways to the afore-mentioned pupil of Scipio del Ferro, Antonio Maria Fior, sometime of Venice. He seems to have turned Ferro's formula to account in the popular mathematical contests then in vogue. Hearing of Tartaglia's claim to solving some cubic, possibly publicized through Da Coi, and thinking Tartaglia an impostor and himself knowing Ferro's solution of $x^3+px=q$, he challenged the latter to a contest. It was set for February 22, 1535. Tartaglia, knowing Fior was only an arithmetician, gave the contest little thought at first. But when he heard that "a great master" "30 years ago" had communicated to him the solution of a cubic, he became worried and set himself to study the equation $x^3+px=q$. (He already had solved $x^3+px^2=q$). On February 14, eight days before the date set for delivering the solutions to the notary who kept the stakes, he found the solution of $x^3+px=q$; and on the next day he also found the solution of $x^3=px+q$.*

Each had challenged the other with thirty questions. As Tartaglia had suspected, all Fior's problems reduced to the form $x^3+px=q$, and he solved them all in two hours. It almost seemed wicked of Tartaglia, for he had constructed problems such that most of them led to the solution of $x^3+px^2=q$ and Fior could not answer a one of them. "I waived the stake and took the honor," says Tartaglia.

Thus ended the second encounter.

We read of these things in the histories. But our modes of life and thinking, our physical environment, are so removed from the 16th century Italy that it is hard for us to reconstruct the tenseness and excitement that accompanied these contests. Honor, gold, and the instinct of game were powerfully present. The questions themselves—the instruments of combat—what did they look like? The histories tell us about $x^3+px=q$. That seems so general and colorless. And then there were no $x^3+px=q$. There were "cube and cose equal to a number" and similar expressions. The challenges did not come in that form either,—they came as problems. And since this is a side-light, we shall see what they are. And, gentle reader, so as to be along in spirit with the tense partisans of that February 22, 1535, solve one or two; you are along in the opening skirmish of the famous "Battle of the Cubic" of the 16th century.

These were the questions submitted by Fior for February 22, 1535:†

(1) Find the number which added to its cube root gives 6.

*Quesito XXV.
†Quesito XXXI.

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(2) Find two numbers in double proportion $[x, 2x]$ such that if the square of the larger is multiplied by the lesser and to the product is added the sum of the numbers, the result is 40.

(3) Find a number which added to its cube gives 5.

(4) Find three numbers in triple proportion $[x, 3x, 9x]$ such that if the square of the smallest is multiplied by the largest and the product be added to the mean number, the result is 7.

(5) Two men were in partnership, and between them they invested a capital of 900 ducats, the capital of the first being the cube root of the capital of the second. What is the part of each?

(6) Two men together gain 100 ducats. The gain of the first is the cube root of the gain of the second. What is the gain of each?

(7) Find a number which added to twice its cube root gives 13.

(8) Find a number which added to three times its cube root gives 15.

(9) Find a number which added to four times its cube root gives 17.

(10) Divide fourteen into two parts such that one is the cube root of the other.

(11) Divide twenty into two parts such that one is the cube root of the other.

(12) A jeweler buys a diamond and a ruby for 2000 ducats. The price of the ruby is the cube root of the price of the diamond. Required the value of the ruby.

(13) A Jew furnishes capital on the condition that at the end of the year he shall have as interest the cube root of the capital. At the end of the year the Jew receives 800 ducats, as capital and interest. What is the capital?

(14) Divide thirteen into two parts such that the product of these parts shall equal the square of the smallest part multiplied by the same.

(15) A person buys a sapphire for 500 ducats and gains the cube root of the capital invested. What was his gain?

(16-30) Deal with lines, triangles, and various equilateral polygons with sides so divided as to become problems of dividing 7, 12, 9, 25, 26, 28, 27, 29, 34, 12, 100, 140, 300, 810, 700 each into two parts such that one is the cube root of the other.

As we see, all these reduce to the form $x^3+px=q$.

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Of Tartaglia's 30 challenge questions to Fior we have record of only the first four. These follow:*

(1) Find an irrational quantity such that when it is multiplied by its square root augmented by 4, the result is a given rational number.
(2) Find an irrational quantity such that when it is multiplied by its square root diminished by 30, the result is a given rational number.
(3) Find an irrational quantity such that when to it is added four times its cube root, the result is thirteen.
(4) Find an irrational quantity such that when from it one subtracts its cube root, the result is 10.

These problems resolve themselves into solving for the irrational quantity x in $x^3+mx^2=n$; $m^2x^2=x^3+n$; $x^3+mx=n$; $x^3=mx+n$.

VI.

It is the ever-moving Da Coi again who brings in the next important personage in these events, Girolamo Cardano. For after his interview with Tartaglia he leaves Brescia and moves to Milano. There he meets Cardan who engages him to instruct one of his classes. Da Coi tells him about Tartaglia and his discovery. Cardan, at this time preparing material for his ambitious work, *Ars Magna,* was much interested in the mathematical duel of Tartaglia and Fior. He therefore sends as messenger Zuan Antonio de Bassano, a book seller, to Tartaglia to inquire about his invention. The atmosphere of the time and the temperament of the principals are well sketched by Tartaglia, under date of January 2, 1539, in

"Quesito XXXI. *Fatto da M. Zuanantonio libraro, per nome d' un Messer Hieronimo Cardano, Medico et delle Mathematice lettor publico in Milano, adi. 2. Genaro, 1539.*"

Zuantonio.—Messer Nicolo, I have been directed to you by a certain man, a good physician in Milano called Messer Hieronimo Cardano, who is a great mathematician and gives public lectures on Euclid in Milano; at present he has a work in press on the art of arithmetic, geometry, and algebra, which will be a beautiful thing.† He has heard of the contest you had with Maestro Antoniomaria Fiore and how

*Quesito XXV.

†Al presente sa stapare una sua opera in la practica Arithmetica et Geometria et in Algebra che sara una bella cosa.

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you agreed to prepare 30 cases or questions each, and that you did that. And his Excellency has heard that all the 30 questions proposed to you by Maestro Antoniomaria led you by algebra to an equation of the cosa and the cube equal to a number (*che ui conduceano in Algebra in un capitolo di cosa e cubo equal a numero*), and that you found a general rule for such an equation and that by the power of this invention you solved in the space of two hours all the 30 cases he proposed to you. On this account his Excellency begs that you would be so kind as to send him this rule that you have invented; and if it pleases you he will insert it in his forth-coming book under your name.

N.—Tell his Excellency that he must pardon me; that when my invention is to be published it will be in my own work. His Excellency must excuse me.

Z.—If you do not want to impart your invention to him, his Excellency ordered me to ask you at least to let him have the 30 above-mentioned cases which were proposed to you together with their solutions. [meaning the results, not the rule obtaining them.]

N.—Not even that can be. For whenever his Excellency observes one of these cases and its solution he will get to understand the rule that I found. And by means of this one rule many others dealing with this subject can be derived." So far, Tartaglia,

After this second rebuff Zuanantonio proposes seven problems leading to these equations:

(1) $2x^3+2x^2+2x+2=10$

(2) $2x^3+2x^2+2x+2=10x$

(3) $2x^3+2x=10$

(4) $2x^3+2x^2=10$

(5) $2x^3+2=10x$

(6) $x^4+8x^2+8^2=10x^3$

(7) $x^3+3x^2=21$

Somewhat hotly Tartaglia rejoins: "These questions are from Messer Zuanne da Coi. And from no one else, for I recognize the last two. Two years ago he proposed to me a question like the sixth and I made him own up that he neither understood the problem nor knew the solution. He also proposed one similar to the last one, which involves working in census and cubes equal to a number [that is, $x^2+px^3=q$] and out of my kindness I gave him the solution less than a year ago. For such solutions I found a particular rule applicable to similar problems."

The bookseller maintains the questions are Cardan's, however. And to support his request he praises Cardan's abilities and deftly

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mentions his connection with a certain high and rich personage, the Marquis del Vasto, a benefactor who was to publish Cardan's book.

"I do not say his Excellency is not a very learned and capable person", says Tartaglia. "But I say he is not able to solve the seven problems which have been proposed to me to be solved by a general rule."

When the messenger leaves he gives him a copy of Fior's 30 questions but not the solutions.

Cardan's reply, February 12, 1539, is full of bitter insult. You are not at the top of the mountain, you are only at its foot, in the valley, he tells Tartaglia, in substance. It is peculiar that you attribute the seven problems to Da Coi, as if there were no one in Milano able to do such a thing. Da Coi is as young as he says he is; I have known him since before he could count to ten. You said if one of these problems is solved, they all are solved. That is completely wrong. I wager 100 ducats you are not able to reduce them to one, nor to two, nor to three equations. (This is the purest invention of Cardan: Tartaglia had said nothing of the kind. Or else the bookseller had misunderstood him.) Concluding, he proposes two problems. The first, taken from Pacioli but not solved by him, requires to find four numbers in geometric progression whose sum is 10 and whose square sum is 60. The second concerns two men in partnership who gain the cube of the tenth part of their several capitals.

To the first Tartaglia in his restrained reply of February 18 gives an elegant solution.* But he is not cajoled into giving away his secret by solving the second, still keeping to himself his solution of "the cube and the cose equal to a number."

Neither tricks nor insults succeeding, Cardan turns to flattery and praise. So in a letter dated March 13, 1539 he begins:† "Messer Nicolo, mio carissimo." Asks Tartaglia not take his former words up in bad part. Blames it onto Da Coi who had given him a wrong idea of Tartaglia. Now the ungrateful wretch has left Milano unceremoniously and also left the sixty pupils he had secured for him. He ends by inviting Tartaglia to visit him in Milano and says that the Marquis del Vasto is anxious to meet him. (This was probably pure fiction.) He concludes the letter with high priase for the nobleman and warm feelings for "mio carissimo" Tartaglia:

"And so I urge you to come at once, and do not deliberate; for the said Marquis is a remunerator of all virtuosi, so liberal and magnanimous that no one who serves his Excellency in any matter remains

*Quesito XXXIII.
†Quesito XXXIII.

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unsatisfied. So do not hesitate to come, and come and live in my house and no otherwheres. May Christ keep you from harm.

March 13, 1539. Hieronimo Cardano, *Physician.*"

This was the rift in the wall that made Tartaglia's citadel crumble. He accepts the invitation and stays a few days in Cardan's house. Their conversation is recorded in Quesito XXXIV under date March 25, 1539:

C.—It is convenient for us that the Marquis has just left for Vigevano so we can talk about our affairs till he returns. You surely have not been any too obliging in not showing me your solutions of the cube and cose equal to a number that I have so earnestly asked you to do.

T.—I tell you, I am niggardly in this matter, not for the sake of this simple equation only and the things that it has enabled me to find, but for the sake of all the things this equation ought to help me discover in the future. For it is a key that opens up the investigation of a great many other equations. If I were not now occupied with the translation of Euclid (I am already on Book XIII) I would already have discovered a general rule for many other equations. [Then the discusses his plan for his book on algebra.] If I now showed the solution to a speculative mind, like your Excellency, he could easily discover the other solutions and publish them as his own, which would completely spoil my project. [Notice all along the distinction between solutions, answers, and the formula or process that gives the solutions.] This is the reason that has compelled me to be so discourteous toward your Excellency; so much the more since you are about to publish a work on a similar subject and in which work, you wrote me, you would like to insert my invention under my name.

C.—But I wrote you that if that did not meet your approval, I will promise to keep it a secret.

T.—As to that, I just can't believe you.

C.—Then I swear you by the holy Evangels of God and as a true man of honor that I will not only never publish it, but I will write it for myself in code so that no one finding them after my death can understand. If you will now believe me, believe; if not, let it pass.

T.—If I did not believe such an oath, I should certainly be regarded as a man without faith. But I have decided to go to Vigevano to find the Marquis; for I have already been here three days and am tired of waiting. On my return I promise to reveal it all.

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C.—If you wish to see the Marquis I will give you a letter so that he may know who you are. But before you go I wish you would show me the rule, as you promised.

Then Tartaglia gives him the solution for $x^3+px=q$ and $x^3+q=px$. Instead of a code Tartaglia gives it in twenty-five lines of poetry, seven tercets followed by a quatrain.* It must have been as good as a code, for in a letter of April 9th† Cardan has trouble with this mathematical poetry. In his reply Tartaglia says it is not $ut=\frac{1}{3}p^3$, but $ut=(\frac{1}{3}p)^3$.

We will just give a taste of this mathematical poetry by quoting one tercet:

> "*Quando che'l cubo con le cose appresso,*
> *Se aggruaglia à qualche numero discreto*
> *Trouan dui altri, differenti in esso.*"

Meaning: If $x^3+px=q$, let $t-u=q$.

The next few lines says: Also let $ut=(\frac{1}{3}p)^3$, then $x=\sqrt[3]{t}-\sqrt[3]{u}$.

Tartaglia must already have begun to feel uneasy, for on leaving Cardan he says to himself: "I will not go to Vigevano. I will go back to Venice, come what may." In the exchange of letters that follow it becomes evident that Cardan is putting his powerful mind to work on Tartaglia's formulae from every angle and soon discerned implications that Tartaglia himself had either not been able to see or was too busy to follow up (he was busy with his translation of Euclid).

The Irreducible Case came up in a letter from Cardan in August, 1539,‡ when Cardan asks: How about

$$x^3=px+q \text{ when } \left(\frac{p}{3}\right)^3 > \left(\frac{q}{2}\right)^2 \text{ as in } x^3=9x+10?$$

Tartaglia saw that Cardan was now making his own investigations and felt none too good about it. He himself could not solve the difficulty, and his answer to Tartaglia lacks frankness. "Has Tartaglia lost spirit maybe from much studying and reading?" banters Cardan§ in his next letter. "If he is sure of understanding the rule he will wager 100 ecus against 25 that he can solve $x^3=12x+20$." Tartaglia did not answer.

On January 5, 1540 came a noteworthy letter from Cardan—noteworthy in the light of what followed.‡‡ Very friendly; not "mio caris-

*For the full Italian version see Cantor's *Geschichte,* (1913), vol. II, pp. 488-9.
†Quesito XXXV.
‡Quesito XXXVIII.
§Quesito XXXIX.
‡‡Risposta I, p. 8.

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simo" now, but "quanto fratello". "That devil" Zuanne dal Colle (as Cardan always spelled it) has returned to Milano and caused him no end of grief. Both in his teaching and in other matters.* But, warns Cardan, he has your equation $x^3+px=q$ and $px+q=x^3$, and he boasts that during his sojourn in Venice he had a discussion with Fior and so arrived at what he searched for. Then he tells of various algebraic solutions that Zuanne had solved, giving details. The whole letter looks like a build-up for 1545, to show that the knowledge of the cubic was not Tartaglia's only.

But Tartaglia does not catch the drift. "Cardan has a mind more dull than I thought", he muses. "Zuanne imposes on him when he says that he has the solution of equations. But I do not want to reply. I have no more affection for him than I have for Messer Zuanne. I will leave them to one another. But I can see that he has lost spirit and does not see how things will turn out."

Then all correspondence between these two ceases.

The next five years were quiet, seemingly. Tartaglia busy with his translations of Euclid and Archimedes, holding in abeyance his future work on algebra; Cardan, assisted by the brilliant Ferrari, working on the *Ars Magna*. In 1545 this monumental work appeared from the Nürnberg press of Petreius. In it, with his consent and under his name, was Ferrari's solution of the biquadratic. In it, and with his name, but not with his consent, was Tartaglia's solution of $x^3+px=q$. The solution that was to have been written in code lest the world should get knowledge of it was broadcast on the pages of Cardan's most ambitious work.

It is given as Chapter XI of the *Ars Magna*,† and is prefaced by a statement that Scipio Ferro had first found the solution, that later Tartaglia also invented it and showed the solution, but not the demonstration, to Cardan.‡ Tartaglia had told Cardan he was jealous of the solution of $x^3+px=q$ not so much for the equation itself, but for the work to which it was the key. And true enough, using this key), ten additional chapters on the cubic besides Ferrari's work on the biquadratic, enrich the contents of the *Ars Magna*. How Tartaglia felt when his eyes saw this, we can imagine. Or can we?

Tartaglia's reply to the statement and the act is given in his *Quesiti*,—documented with names, circumstances, dates, places,—published the following year. Cardan never satisfactorily met those

*Quesito XXXX.

†A translation of this chapter, with comments, by R. B. McClenon is found in D. E. Smith's *Source Book of Mathematics*, New York, 1929, pp. 203-6.

‡The edition examined for this article was this same first edition. There were some changes in later editions.

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accusations in writing, nor could Tartaglia entice him to meet him in person for a mathematical combat.

VII.

Now comes a most unique spectacle in mathematical history; not as mathematics but as human passions, wickedness, and contrariness.

Cardan did not reply to the accusations of Tartaglia. But Ludovico Ferrari, his grateful pupil, "suo creato", took up the gauntlet for his master. On February 10, 1547, he sent a public challenge to Tartaglia at Venice: a pamphlet with four pages of content and four (!) pages of names of mathematicians in various universities and cities to whom copies of the challenge had been sent, fifty in all. Among these we notice the name of Ferro's successor, "Hannibal dalle Nave"; but neither Da Coi nor Fior.

"Messer Nicolo Tartaglia", it begins, "there has come into my hands a book by you called *Quesiti ed inventioni nuovi,* in the last treatise of which you mention his Excellency Signor Hieronimo Cardano, a physician at Milano, who is at present a public lecturer in medicine at Pavia. And you are not ashamed to say that he is ignorant in mathematics, that he is a dull individual, deserving to have Messer Giovan da Coi placed before him.................................
I think you have done this, knowing that Signor Hieronimo has such a great genius that not only in medicine, which is his profession, has he a reputation for ability, but also in mathematics, in which he has at times indulged as a game, to get recreation and enjoyment, and in which he has become so widely successful that without exaggeration he is considered one of the great mathematicians."

Besides a multitude of errors, the challenge continues, Tartaglia has also plagiarized from Jordanus, whose propositions he has placed in the 8th book without citing his name.* Tartaglia has blamed Cardan unjustly; he, who is not worthy to mention Cardan's name (*il quale à pena sete degno di nominare*).† Thereupon he challenges Tartaglia to a public disputation from ancient and modern authors on "Geometry, Arithmetic, and all the disciplines that depend on these, as Astronomy, Music, Cosmography, Perspective, Architecture, and others,................, and not only on what Latin, Greek and "vulgar" [vernacular, modern] authors write on these subjects, but also on your own inventions."

The time was thirty days; the stake, up to 200 *scudi,* to be decided by Tartaglia. "And in order that this invitation shall not appear too

*Cartello I, p. 2.
†Cartello I, p. 5.

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private, I have sent a copy of this writing to each of the gentlemen named below."

Thus begins the fourth part of this celebrated controversy.

Tartaglia replies on February 19, nine days later. Also a printed pamphlet. Equally formal: "From Nicolo Tartaglia of Brescia, Professor of mathematics in Venice, to Messer Ludovico Ferraro, Public Lecturer of Mathematics in Milano." Six pages of compact print, with four witness signatures. But instead of an impressive list of mathematicians at the end, he has a postscript:*

"And in order that this reply of mine shall not appear too private, I have had 1000 copies printed to send them around Italy in general; since I am not acquainted with the cities in Italy or with the universities, where one can buy the friendship and knowledge of experts and scholars, as you do (because, in truth, my experience and acquantance are limited to my study and to my students). For this reason I do not only not have the friendship but not even the acquaintance of these persons."

Therefore, he continues, he will not send his reply to the persons Ferrari names; could not do it even to a certain named person, "for he died two months ago." (Ferrari's challenge had come nine days ago!)

As for a response, he will not meet Ferrari in combat. It is with Cardan he has a quarrel, and when that gentleman is ready, Tartaglia will accept. So he sends a counter-challenge that Cardan and Ferrari on one side and he on the other submit one to the other a list of problems to solve.

Six weeks later, on April 1, comes a second challenge from Ferrari,— *and this time in Latin.* Why change from Italian to Latin? Tartaglia thought he knew.

In Cartello II Ferrari touches upon the solution of the cubic equation.† Tartaglia has taken umbrage at Cardan for publishing his solution of the cubic. What if the published solution was that of a third party? Five years ago, declares Ferrari, in 1542, Cardan and Ferrari were in Bologna and there visited Annibale della Nave, Scipio Ferro's son-in-law, who showed them books written by Scipio; and there was the solution Cardan published. Annibale is alive today and can be called as a witness anytime. (This would be a serious argument, except it is not convincing. Why, since there was no secrecy about it, had Cardan in *Ars Magna* not mentioned Scipio's *writings* and Anibale's part, instead of referring only to Scipio and Fior?)

A large portion of Tartaglia's replies is sparring for objectives. He regularly wants to get into combat with Cardan himself; just as

*Risposta I, p. 8.
*Cartello II, p. 2.

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regularly the slippery Ferrari turns him off. Another objective is to have the contest be a list of challenge questions, to be solved in a specific time; Ferrari wants a public disputation in Rome, Florence, Pisa, or Bologna, to be chosen by Tartaglia, and judges to be selected from persons in the city chosen.

Two points taken up in Risposta II evoke our sympathy:

The first concerns the mode of contest. Besides considering the method of challenge lists a better arbiter of ability than public disputations the sender may have had an added reason for his choice. Tartaglia, "the stammerer", had an impediment of speech ever since as a child he was cut by a French soldier in the cathedral massacre at Brescia. In a public disputation he would have a serious handicap engaging the oily Cardan and the brilliant Ferrari.

The second dealt with where and with whom the stake was to be deposited. *And if only gold was to be used*, or whether Tartaglia could, for part of the sum, deposit printed copies of the *Quesiti*. Remembering Tartaglia's worldly circumstances one can here read much between the lines.

Replying to the charge of plagiariam, he says:* Though the statement of the propositions emanated from Jordanus, the demonstrations and arrangement were Tartaglia's. The statement of a proposition without the proof is of no value. Answering Ferrari's reference to the cubic:† "It would be presumptous of me to give the impression that the result which I discovered could not also have been discovered at other times and by other persons, and that they cannot likewise be discovered in the future and by other persons: even when they will not be given to the public by Signor Hieronimo or myself. But this can I say with truth, that I never saw these things in any author but discovered it myself."

He pokes a thrust at Cardan for not being willing to enter the contest but sending Ferrari instead. And then he dishes up this pretty one:

"You say that you have heard that in the past few years I have made machines and invented several types of instruments and that people think that by my persistent knowledge I have succeeded in making a machine with which I can shoot clear to Milano while I am stationed in Venice.

"Regarding this particular, I answer they are not at all wrong. For since the presentation of your Cartello I have actually built one with which, while I am in Venice, I can shoot, not only as far as Milano,

*Risposta II, pp. 7-8.
†Risposta II, p. 6.

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but even as far as Pavia [Cardan had left Milano and was now located in Pavia], and shoot with such a direct aim that I will not only scare you and Signor Hieronimo but cause you great anguish."*

Then he proposes a challenge of thirty one questions.† He states that he can solve them, adding: "I am not like Signor Hieronimo who presents cases he does not know how to solve himself."

Ferrari counters with 31 other questions in the Cartello III (June 1) without answering Tartaglia's. This letter is a highly insulting piece. "In response to my reply," he says, "I have received your *tartagliata* [a pun on the etymology of Tartaglia's name]: which, though long and confused, contains nothing but insults, refusals to admit defeat, and a fixed idea of wanting to fight while running away."

In Risposta III (July 9) Tartaglia gives the solution to Ferrari's 31 questions and boasts he is the victor. In Cartello V (October, 1547) Ferrari tears Tartaglia's answers to pieces mercilessly and claims only five are correct. This Cartello is almost a book in size, all of 55 pages; 41 page are mathematics and contain Ferrari's solutions of Tartaglia's challenge list.

Eight months pass before the answer comes, the longest time between any of these exchanges. And then the unlooked-for happens: on June 1, 1548, Tartaglia accepts the challenge to a public disputation and even to have it in Cardan's and Ferrari's own bailiwick, Milano.‡ How and why the change? He may have become so exasperated with Ferrari's manhandling of his solutions in Cartello V and of other provocative matters in the Cartello—and he did want a duel with Cardan—that he was willing to forego his preferred mode of question lists. Or there may have been other reasons as insinuated by Ferrari, necessitating "Brescia *versus* Milano."§ For city pride and championships did not begin in the 20th century. Tartaglia had recently moved to Brescia where some of its chief citizens had promised him liberal remuneration for giving public lectures on Euclid. Ferrari insinuates that his acceptance of the challenge to Milano was one of the stipulations.

Tartaglia accepted the challenge. But let no one think there was sportsman's etiquette there. Cartello VI (July 14, 1548) and Risponta VI (July 24, 1548) perforce takes up arrangements on the business element. But they have plenty of room for smarting sentences, especially Ferrari's. He seems to have fed on his own anger and vindictiveness.

*Risposta II, p. 11.
†Risposta II, p. 15-20.
‡Risposta V, p. 7.
§Cartello VI, p. 9.

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For a year and a half these tirades had continued. On August 10, 1548, the "disputation" took place, and with what outcome we read in Tartaglia's own account, written* nine years later, in an article interpolated in his *General Trattato.*

It follows:

"In 1547 Cardan and his creature Ludovico Ferraro brought me a challenge in two printed pamphlets. I addressed to them 31 questions, with the stipulation that they should be solved in 15 days after receiving them. After that the solution should be considered as not arrived. They let two months pass without giving any sign of existence, and then they sent me 31 questions without giving me the solutions of any of mine; besides, the term stipulated had passed by more than 45 days. I found the solution of 10 of them the same day, the next day a few more, later all the rest, and, so as not to let pass the interval of 15 days, I hurried to get them printed and sent to Milano. In order to conceal their slowness in answering my questions or at least a few of them they took up my time with other matters full of silly foolishness, and it was not till the end of seven months that they sent me a public reply where they boasted that they had solved my questions. However, even had that been true, those solutions given so long a time after the term fixed would have been without any merit; but the greater part of them were completely wrong. I desired to proclaim publicly that they were wrong, so, being in Brescia and hence in the neighborhood of Milano, I sent to them both a printed challenge asking them to meet me the following Friday, August 10, 1548 at 10 10 o'clock near the church called the Garden of the Order of Zoccolante to argue publicly my refutation of their pretended solutions. Cardan, so as not be at the examination, left Milano hurriedly.

"On the day set Ferraro came alone to the meeting-place, accompanied by a crowd of friends and by many others. I was alone with my brother whom I had taken along from Brescia. I went before the entire crowd and began by giving briefly an exposition of the subject for discussion and the reason for my arrival in Milano. When I wanted to take up the refutations of the solutions I was interrupted for a period of two hours by words and actions under pretext that there should be chosen, at that very place, a certain number of judges from the auditors present, all friends of Ferraro and to me entirely unknown. I would not consent to such a trick and said that my understanding was that all the auditors were judges, the same as those who read my refutation when printed. Finally they let me speak, and in order not

*Part II, Book II, Chapter, 7, article 7.

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to tire my audience I did not commence with tedious topics from number theory and geometry, but it seemed to me suitable to refute their solution of a question in Chapter 24 in Ptolemy's Geography; and I constrained Ferraro publicly to own that he was in error. When I wished to continue they all began to shout that now I should discuss my own solutions of the 31 questions that had been proposed to me and which I had solved in 3 days. I objected that they should let me finish what concerned my refutations, then I would take up what they asked for. Neither reasoning nor complaining could be heard. They would not let me speak further and gave the word to Ferraro. He began by saying I had not been able to solve the fourth question in Vitruvius, and he expatiated on this clear till the supper hour. Then everybody left the church and went home."

So far Tartaglia. Seeing the temper of the crowd and fearing violence, he did not wait to continue his disputation the next day but hurriedly left for Brescia by another road than that by which he had come, glad to keep his life.

So ended this combat at the Church of Zoccolante, original even for this age.

VIII.

In 1556, (ten years after the appearance of *Quesiti*), came the first two parts of Tartaglia's great life work, for the contents of which he had reserved many discoveries made even before the *Inventioni* of 1546.

It was the *General Trattato di numeri, et misure,* a huge, ambitious and well-written work on arithmetic and algebra. It is said to be the best work on arithmetic in the entire century. The third part, which was not published till 1560, was left uncompleted; for Tartaglia died in 1557. It was largely algebra and it is thought the last division was to have included his work on the cubic equation. As it is, we have only so much of his work in this field as is found in the *Quesiti* of 1546.

The mystic equation:

$$e^{i\pi}+1=0$$

is indeed awe-inspiring. Here, solidly welded together, are the important representatives of the *real,* the *complex,* and *natural* number system.

$i^i=e^{-\pi/2}$, a real!

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